Dominant Pole Synthesis of Transmission Line Networks

SOLAIMANUL MAHDI, STUDENT MEMBER, IEEE, AND ALAN B. MACNEE, FELLOW, IEEE

Abstract—**This paper describes a procedure for synthesizing transmission networks which are interconnections of uniform line elements. An iterative, digital computer algorithm is developed which achieves a dominant pole synthesis. The line lengths and the characteristic impedances are controlled individually, which gives design flexibility not found in synthesis procedures based on Richards' transformation. Thus, the characteristic impedances may be restricted by upper and lower bounds when there is no restriction on the line lengths. The procedure is detailed for a TEM mode structure of alternating open stubs and connecting lines. The method uses a Newton-Raphson iterative scheme to adjust the characteristic impedances and lengths of the transmission lines for a prescribed set of dominant transmission poles. By controlling the stub line lengths and the dominant pole positions, the principal transmission zeros and bounded characteristic impedances can be achieved simultaneously.**

I. Introduction

NETWORK functions that characterize a lossless transmission line section can be expressed in terms of a single function tanh $(s/4f_0)$, where $s=\sigma+j\omega$ is the complex frequency, and f_0 is the frequency, in hertz, at which the length of the transmission line element is a quarter wavelength. Using the transformation $\lambda=\Sigma+j\Omega=\tanh(s/4f_0)$ Richards [1] showed that distributed circuits composed of lumped resistors and equal length transmission line elements can be treated exactly as lumped networks in the new variable λ. Thus all the power of conventional lumped parameter synthesis techniques is made available to the designers of distributed parameter networks.

Richards' transformation has been successfully employed for the analytical design of distributed networks by Ozaki and Ishii [2], Horton and Wenzel [3], Wenzel [4], and many others. While this transformation has been a powerful tool for the analytic design of distributed structures, techniques based on it have certain weaknesses. Foremost among these is the requirement that all the transmission line elements have the same electrical length. One-half of the available degrees of freedom are being fixed for analytic convenience, and all of the design control rests in the characteristic impedances of the transmission line elements. This can lead to practical difficulty since the range of feasible characteristic impedance is much less than the element value range available with lumped inductances and capacitances.

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The authors are with the Department of Electrical Engineering, University of Michigan, Ann Arbor, Mich.

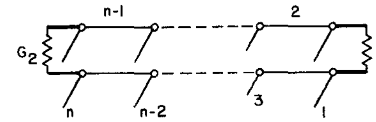


Fig. 1. General structure of the transmission line networks considered in this paper. The length l_j and the characteristic impedance z_j characterize the jth transmission line.

A microwave network structure with no restriction on line lengths has been considered by Kinariwala [5]. He has derived necessary and sufficient condition for a multivariable function to be the input impedance of a circuit made up of sections of unequal length transmission lines in cascade and terminated in a lumped resistor. He also gives a procedure for the synthesis of such a cascaded structure from an input impedance satisfying the given conditions. The approximation problem for such a cascade network is still unsolved, as are the synthesis and approximation methods for a more general structure.

Although analytic design methods for distributed networks without the a priori restriction of commensurable line lengths are yet to be found, the high-speed computational capability of a large digital computer makes feasible the iterative design of such structures.

This paper describes a dominant pole synthesis procedure applicable to a distributed TEM mode network having the physical structure indicated in Fig. 1; in this procedure the lower and upper bounds of the characteristic impedances are specified, but there is no restriction on line lengths. The method utilizes a Newton-Raphson iterative scheme to adjust the characteristic impedances and lengths of the transmission lines for a prescribed set of dominant transmission poles. If the poles are chosen to give a low-pass characteristic when all of the transmission zeros are at infinity, the finite transmission zeros produced by the stubs in Fig. 1 may substantially modify the transfer characteristic realized. By controlling the stub line lengths and modifying the dominant pole positions, improved transmission characteristics and bounded characteristic impedances can be achieved simultaneously.

II. The Analytic Problem

The characteristic impedances and the lengths of the transmission line elements that realize a prescribed set of transmission poles are the solution of a system of nonlinear equations. In this section, this system of nonlinear equations is derived.

The general circuit parameter matrices of a connecting line element and an open stub element are respectively given by

$$\begin{bmatrix} \cosh \frac{sl}{v} & z \sinh \frac{sl}{v} \\ \frac{1}{z} \sinh \frac{sl}{v} & \cosh \frac{sl}{v} \end{bmatrix}, \tag{1}$$

and

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{z} \tanh \frac{sl}{v} & 1 \end{bmatrix} = \frac{1}{\cosh \frac{sl}{v}} \begin{bmatrix} \cosh \frac{sl}{v} & 0 \\ \frac{1}{z} \sinh \frac{sl}{v} & \cosh \frac{sl}{v} \end{bmatrix}. \tag{2}$$

In matrices (1) and (2), l, z, and v denote, respectively, the transmission line length, the characteristic impedance, and the velocity of propagation. The general circuit parameter matrix for the whole circuit in Fig. 1 is the product of individual matrices representing each circuit element, which may be written

$$\begin{bmatrix} 1 & 0 \\ G_2 & 1 \end{bmatrix} \left\{ \frac{1}{\cosh \frac{sl_n}{v}} \begin{bmatrix} \cosh \frac{sl_n}{v} & 0 \\ \frac{1}{z} \sinh \frac{sl_n}{v} & \cosh \frac{sl_n}{v} \end{bmatrix} \right\} \begin{bmatrix} \cosh \frac{sl_{n-1}}{v} & z \sinh \frac{sl_{n-1}}{v} \\ \frac{1}{z} \sinh \frac{sl_{n-1}}{v} & \cosh \frac{sl_{n-1}}{v} \end{bmatrix} \cdots \begin{bmatrix} \cosh \frac{sl_2}{v} & z \sinh \frac{sl_2}{v} \\ \frac{1}{z} \sinh \frac{sl_2}{v} & \cosh \frac{sl_2}{v} \end{bmatrix}$$
$$\cdot \left\{ \frac{1}{\cosh \frac{sl_1}{v}} \begin{bmatrix} \cosh \frac{sl_1}{v} & 0 \\ \frac{1}{z} \sinh \frac{sl_1}{v} & \cosh \frac{sl_1}{v} \end{bmatrix} \right\} \begin{bmatrix} 1 & 0 \\ G_1 & 1 \end{bmatrix} = \begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} = \prod_{i=1,3,\cdots}^{n} \left[\operatorname{sech} \left(\frac{sl_i}{v} \right) \right] \begin{bmatrix} A_T' & B_T' \\ C_T' & D_T' \end{bmatrix}. \tag{3}$$

Since the transfer impedance is given by

$$z_{12} = \frac{1}{C_T} = \frac{\prod_{i=1,3,\cdots}^{n} \left[\cosh \left(\frac{sl_i}{v} \right) \right]}{C_T'},$$

transmission poles are the zeros of C_T'. To emphasize that C_T' is a function of lengths of the line elements, $(l_1, l_2, \cdots, l_n)$, the characteristic impedances of the line elements, $(z_1, z_2, \cdots, z_n)$, and the complex frequency s, it can be written as

$$C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; s).$$

If ξ (and its conjugate) is a complex transmission pole of the network in Fig. 1, one gets

$$C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; \xi) = 0. \tag{4}$$

C_T' being a complex-valued function, (4) can be separated into two real equations:

$$\operatorname{Re} [C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; \xi)] = 0, \tag{5}$$
$$\operatorname{Im} [C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; \xi)] = 0. \tag{6}$$

If ξ is real, C_T' is real, and in that case (4) is a real equation. A complex pole and its conjugate result in two equations, and a real pole results in one equation. Realization of n transmission poles at prescribed locations s_i, $i = 1, 2, \cdots, n$ requires that

$$C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; s_i) = 0; \quad i = 1, 2, \cdots, n. \tag{7}$$

Following the procedure outlined above the system of equations (7) may be written in the form

$$f_i(l_1, \cdots, l_n; z_1, \cdots, z_n) = 0; \quad i = 1, \cdots, n. \tag{8}$$

The functions f_i are the real and imaginary parts of C_T' at the complex poles and C_T' itself at the real poles. Assuming that the prescribed set of n pole locations are given by

$$s_1, s_2 = s_1^*; \quad s_3, s_4 = s_3^*; \cdots;$$
$$s_{n-2}, s_{n-1} = s_{n-2}^*; \quad s_n(\text{real}),$$

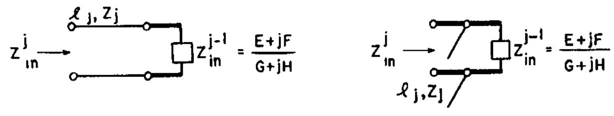
(8) can be explicitly expressed as

$$\begin{aligned} f_1 &= \operatorname{Re} [C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; s_1)] = 0 \\ f_2 &= \operatorname{Im} [C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; s_1)] = 0 \\ f_3 &= \operatorname{Re} [C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; s_3)] = 0 \\ f_4 &= \operatorname{Im} [C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; s_3)] = 0 \\ &\vdots \\ f_{n-2} &= \operatorname{Re} [C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; s_{n-2})] = 0 \\ f_{n-1} &= \operatorname{Im} [C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; s_{n-2})] = 0 \\ f_n &= C_T'(l_1, \cdots, l_n; z_1, \cdots, z_n; s_n) = 0. \end{aligned} \tag{9}$$

III. Calculation of Real and Imaginary Parts of C_T'

C_T' may be calculated by multiplying the component matrices in (3). Another way of finding C_T' is calculating the input impedance at the port 1-1′ in Fig. 3, since $z_{\text{in}} = A_T'/C_T'$. Evaluation of z_{in} proceeds in the following way.

The input impedance of a transmission line terminated in an impedance $(E+jF)/(G+jH)$ as indicated in Fig. 2(a), can



(a) (b)

Fig. 2. (a) A transmission line terminated by the impedance $z_{\text{in}}^{j-1}=(E+jF)/(G+jH)$. (b) An impedance $z_{\text{in}}^{j-1}=(E+jF)/(G+jH)$ shunted by an open stub.

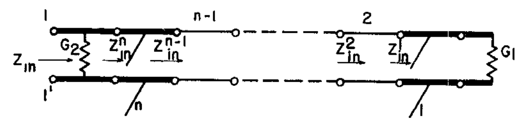


Fig. 3. Input impedance z_{in} looking into the port $1-1'$ is determined recursively. Computation starts at G_1, then proceeds towards the left as $z^j(j=1, 2, \ldots, n)$ are found using (10) or (11), until port $1-1'$ is reached.

be manipulated into the form

$$z_{\text{in}}{}^{j}=z_j\frac{(E+jF)\cosh(sl_j/v)+(G+jH)z_j\sinh(sl_j/v)}{(G+jH)z_j\cosh(sl_j/v)+(E+jF)\sinh(sl_j/v)}. \quad (10)$$

The input impedance of a one port shunted by an open stub as depicted in Fig. 2(b) is found to be

$$z_{\text{in}}{}^{j}=\frac{(E+jF)z_j\cosh(sl_j/v)}{(G+jH)z_j\cosh(sl_j/v)+(E+jF)\sinh(sl_j/v)}. \quad (11)$$

Starting with $E=1/G_1$, $F=0$, $G=1$, $H=0$, $z_{\text{in}}{}^1, z_{\text{in}}{}^2, \cdots, z_{\text{in}}{}^n$ are computed recursively by using whichever of the expressions (10) or (11) is applicable as shown in Fig. 3.

Finally z_{in} is given by

$$z_{\text{in}}=\frac{1}{G_2+(1/z_{\text{in}}{}^n)}. \quad (12)$$

Evaluation of C_T' by first finding z_{in} from (12) involves determination of $z_{\text{in}}{}^j(j=1, 2, \cdots, n)$ using either (10) or (11). Each of these $z_{\text{in}}{}^j$ is expressed by four real numbers. In the computer program that was written to solve the system of equations (9), C_T' was found from the denominator of z_{in} rather than from the alternate way of taking the product of matrices in (3), since representing each matrix requires eight real numbers.

IV. Approximate Solution

A method of evaluating an approximate solution of the system of equations (9) under suitable assumptions regarding the lengths and the characteristic impedances is outlined in this section.

Using the series expansions for $\cosh(sl/v)$ and $\tanh(sl/v)$, the transmission matrices (1) and (2) for a connecting line and an open stub can be written in the forms

$$\begin{bmatrix} 1+\frac{l^2}{v^2}\frac{s^2}{2!}+\cdots & z\left(\frac{l}{v}s+\frac{l^3}{v^3}\frac{s^3}{3!}+\cdots\right) \\ \frac{1}{z}\left(\frac{l}{v}s+\frac{l^3}{v^3}\frac{s^3}{3!}+\cdots\right) & 1+\frac{l^2}{v^2}\frac{s^2}{2!}+\cdots \end{bmatrix}, \quad (13)$$

and

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{z}\left(\frac{l}{v}s-\frac{l^3}{v^3}\frac{s^3}{3}+\cdots\right) & 1 \end{bmatrix}. \quad (14)$$

If for the connecting line it is assumed that

$$z\to\infty, \quad l/v\to 0, \quad (15)$$

and

$$zl/v\to L,$$

the matrix (13) in the finite frequency-plane approaches the limiting value

$$\begin{bmatrix} 1 & sL \\ 0 & 1 \end{bmatrix}. \quad (16)$$

Similarly if the open stub satisfies these conditions:

$$z\to 0, \quad l/v\to 0, \quad (17)$$

and

$$\frac{l/v}{z}\to C,$$

the matrix (14) in the limit becomes

$$\begin{bmatrix} 1 & 0 \\ Cs & 1 \end{bmatrix}. \quad (18)$$

Therefore, in Fig. 3, if all the connecting lines satisfy conditions similar to (15), i.e.,

$$z_j\to\infty, \quad l_j/v\to 0, \quad (19)$$

and

$$z_jl_j/v=L_j, \qquad j=2, 4, 6, \cdots$$

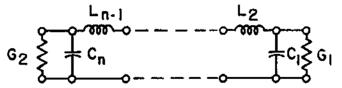


Fig. 4. Approximate lumped equivalent of the distributed network in Fig. 3 under the assumptions (19) and (20).

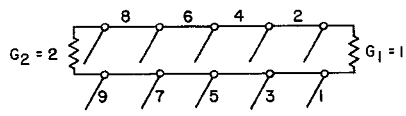


Fig. 5. A nine-element transmission line network.

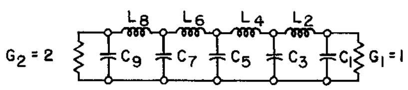


Fig. 6. The lumped network having ninth-order Butterwoith transmission poles. The element values are given in column 1 of Table I.

and all the open stubs satisfy conditions similar to (17), i.e.,

$$z_j \rightarrow 0,$$

$$l_j/v \rightarrow 0,$$

and

$$\frac{l_j/v}{z_j} = C_j, \qquad j = 1, 3, 5, \cdots \tag{20}$$

the general circuit matrix for the lossless portion of the distributed structure in Fig. 3 becomes

$$\begin{bmatrix} 1 & 0 \\ C_n s & 1 \end{bmatrix} \begin{bmatrix} 1 & L_{n-1}s \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & L_2 s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ C_1 s & 1 \end{bmatrix}. \tag{21}$$

The matrix (21) is realized as the lossless part of the lumped network in Fig. 4. Assuming that all the connecting lines have very high characteristic impedances, all the shunt stubs have very low characteristic impedances, and all the line lengths are electrically short, the distributed network in Fig. 3 can be approximately represented by the lumped network in Fig. 4. It follows that the poles of this lumped network are very close to the dominant transmission poles of the distributed structure.

To obtain an approximate solution of the system of equations (9), the lumped network in Fig. 4 is first synthesized for the prescribed set of transmission poles. With the values of inductances and capacitances thus determined, the lengths of the transmission line in Fig. 3 are found from the relations

$$l_j/v = \frac{L_j}{z_j}, \qquad j = 2, 4, 6, \cdots \tag{22}$$

$$l_j/v = C_j z_j, \qquad j = 1, 3, 5, \cdots \tag{23}$$

using arbitrarily chosen high values of characteristic impedances for connecting lines and low values of characteristic impedances for the stubs. For the examples that were worked out by the writers the values of 10 for the impedances of the connecting lines and 0.1 for the impedances of the stubs were used in determining the lengths of the lines. This gave a good approximate solution of the system of equations (9) when the terminations were of the order of unity.

TABLE I

		Approximate Solution from (22) and (23)	Solution Refined by Newton–Raphson Method
$C_1=3.0223$	$z_1=0.1$	$l_1/v=0.30223$	0.3536814
$L_2=0.9579$	$z_2=10.0$	$l_2/v=0.09579$	0.09084188
$C_3=3.7426$	$z_3=0.1$	$l_3/v=0.37426$	0.3598843
$L_4=0.8565$	$z_4=10.0$	$l_4/v=0.08565$	0.08299981
$C_5=2.9734$	$z_5=0.1$	$l_5/v=0.29734$	0.2877055
$L_6=0.6046$	$z_6=10.0$	$l_6/v=0.06046$	0.05878263
$C_7=1.7846$	$z_7=0.1$	$l_7/v=0.17846$	0.1730608
$L_8=0.2735$	$z_8=10.0$	$l_8/v=0.02735$	0.0265765
$C_9=0.3685$	$z_9=0.1$	$l_9/v=0.03685$	0.03653245

The capacitances and the inductances are given, respectively, in farads and henrys. Characteristic impedances are in ohms and delays l/v are in seconds.

As an example, consider the 9-element distributed structure in Fig. 5. It is required to find the approximate values of the lengths and the characteristic impedances [i.e., the approximate solution of the system (9)] that realizes the ninth-order Butterworth poles as the dominent transmission poles of this structure. The lumped network that has these prescribed transmission poles (and all the transmission zeros at infinity) is shown in Fig. 6.

The element values of the distributed network found by using (22) and (23) and the values of the inductances and capacitances for the lumped network are given in Table I.

V. Refinement of the Solution by Newton-Raphson Method

Once an approximate solution is obtained, this can be refined to any desired degree of accuracy by the Newton-Raphson [6] technique. Observe that the system of equations (9) consists of n equations but possesses $2n$ degrees of freedom. Therefore this system can be solved for any n of these $2n$ variables (n line lengths and n characteristic impedances), the remaining n variables being assigned arbitrary values. For example, if the characteristic impedances are picked arbitrarily, the system (9) is solved for the lengths of the lines. Thus system (9) reduces to

$$f_i(l_1, \cdots, l_n) = 0, \qquad i = 1, 2, \cdots, n. \tag{24}$$

This can be solved by the Netwon-Raphson method starting with approximate solution obtained by the procedure outlined in Section IV. The result of applying this iteration method to the approximate solution of the 9-element distributed network in Fig. 5 is given in column 4 of Table I.

VI. Realization of Characteristic Impedances Within a Prescribed Bound

In the initial solution of the system of equations (9) outlined in Section V, the characteristic impedances of the connecting lines are impractically high and the characteristic impedances of the stubs are impractically low. However, this

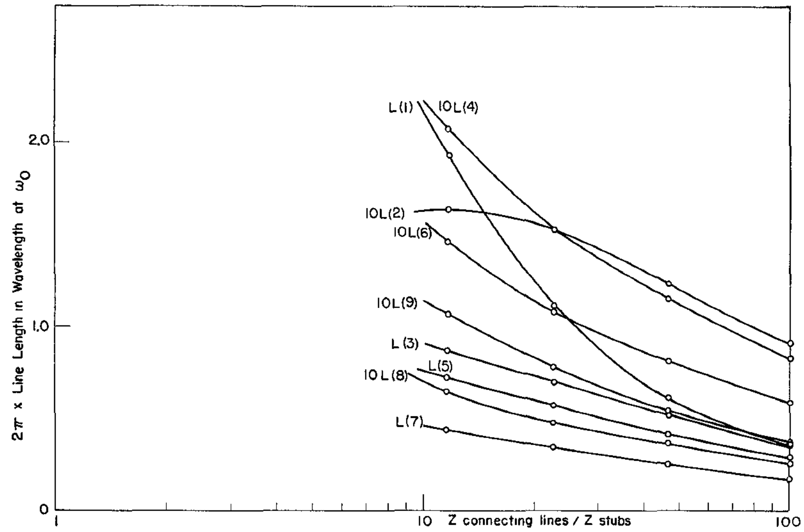


Fig. 7. Variation of line lengths necessary to maintain Butterworth poles ($n=9$) as the ratio of $z_{\text{connecting lines}}$ to z_{stub} is reduced.

difficulty can be overcome. The characteristic impedances of the connecting lines can be gradually lowered and the characteristic impedances of the stubs can be gradually increased, and each time a change in the impedance (or impedances, in case more than one impedance is changed simultaneously) is made, the system (9) is solved again to find new values of lengths of the line elements, thus preserving the prescribed dominant poles. The iteration for this new solution is initiated with the old solution as the starting point. The variation of line lengths necessary to maintain the prescribed Butterworth poles for the 9-element filter as the ratio of characteristic impedance of connecting lines to that of the stubs was decreased is graphically presented in Fig. 7.

Decreasing the ratio $z_{\text{connecting lines}}/z_{\text{stubs}}$ results, in general, in an increase of the line lengths. When all the connecting lines have characteristic impedances of 3.5 ohms and all the stubs have characteristic impedances of 0.3 ohms, the line lengths that realize the Butterworth transmission poles were found to be

$$\begin{aligned} l_1/v &= 1.933716 \text{ second} & l_6/v &= 0.1454346 \text{ second} \\ l_2/v &= 0.1635795 \text{ second} & l_7/v &= 0.4402347 \text{ second} \\ l_3/v &= 0.8731836 \text{ second} & l_8/v &= 0.06485635 \text{ second} \\ l_4/v &= 0.204529 \text{ second} & l_9/v &= 0.1061295 \text{ second.} \\ l_5/v &= 0.7253457 \text{ second} \end{aligned} \tag{25}$$

The computer program written for changing the characteristic impedances has the capability of automatically bringing down the ratio $z_{\text{connecting line}}/z_{\text{stub}}$ while finding the lengths of the transmission lines needed to maintain the transmission poles at the prescribed positions. A typical run is initiated by specifying the prescribed poles, the approximate solution, and that the line lengths are to be varied.

This program has been successfully applied to distributed structures having 2, 3, 5 and 9 elements when the transmission poles were prescribed to be the Butterworth poles. In each of these cases, impedances of the connecting lines were each decreased from 10 to 3.333 ohms and the impedances of the stubs were each increased from 0.1 to 0.3 ohms automatically.

Although the distributed networks obtained through the above procedure realize the prescribed set of dominant transmission poles, the increased lengths of the line elements, which accompany the decreased impedance ratio, produce additional effects. The finite lengths of the stubs produce transmission zeros at frequencies for which the line lengths are odd multiples of one-quarter wavelength. Thus a shunt stub of length l produces transmission zeros at frequencies

$$\omega_k = \frac{k\pi/2}{l/v} \qquad k = 1, 3, 5, \cdots. \tag{26}$$

Such transmission zeros can be used to improve stop-band attenuation. However, to preserve the passband attenuation characteristic in the presence of transmission zeros, the dominant transmission poles must be moved to new locations.

Consider, as an example, the 9-element filter in Fig. 5 with the element values (25). If it is assumed that only the first transmission zeros produced by each of the stubs 1, 3, 5, and 7 are significant, 8 finite transmission zeros must be considered. For example, by the application of the theory of inverse Chebyshev [7] filters, it is found that for the passband response to be maximally flat, pole locations should be shifted from the Butterworth locations to

$$\begin{aligned} S_{1,2} &= -\ 0.112954 \pm j\ 1.002688 \\ S_{3,4} &= -\ 0.37495 \pm j\ 1.016385 \\ S_{5,6} &= -\ 0.749856 \pm j\ 0.984713 \\ S_{7,8} &= -\ 1.257454 \pm j\ 0.716270 \\ S_9 &= -\ 1.565015 \pm j\ 0, \end{aligned} \tag{27}$$

and eight zeros should be placed at

$$\begin{matrix} \pm j\ 1.320055, & \pm j\ 1.501110, \\ \pm j\ 2.022441, & \pm j\ 3.800946. \end{matrix} \tag{28}$$

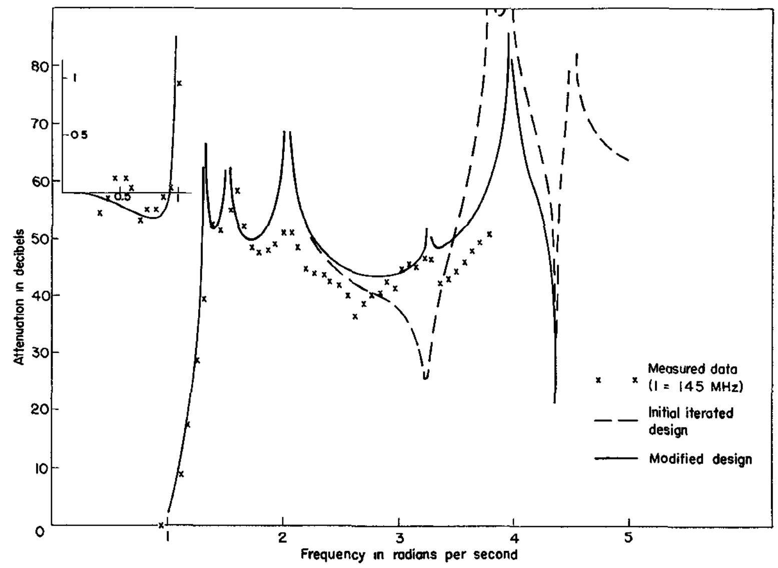


Fig. 8. Calculated and measured attenuation characteristics of nine-element stripline filter.

In this design the transmission zeros are placed to produce equiripple response in the stop band. It should be noted that because of nondominant transmission poles and zeros of the distributed system, which are not accounted for in this lumped design, (27) represents approximations to the dominant transmission pole locations desired in the distributed design.

The lengths of the stubs required to produce the transmission zeros (28) are calculated to be 1.19, 1.047, 0.775, and 0.4135 second. To attain the desired transmission zeros given by (28) the lengths of the stubs (1), (3), (5) and (7) were gradually changed from their values 1.933716, 0.8731836, 0.7253457, 0.4402347, to respectively 1.19, 1.047, 0.775, 0.4135. While changing the lengths of these stubs, appropriate changes were made in the remaining variables [nine characteristic impedances and the length of the four connecting lines and the stub (9)] to preserve the Butterworth poles. In the computer program written to achieve this, any n of the $2n$ parameters defining the system can be kept fixed, while the iteration is carried out on the remaining n parameters. In this particular example, while lengths of the stubs were gradually changed to desired dimensions, iteration was performed on z_1, l_2, z_3, l_4, z_5, l_6, z_7, l_8, and z_9. The element values that achieves the transmission zeros (28) and maintains the Butterworth transmission poles were found to be

$l_1/v = 1.19$	seconds	$z_1 = 0.1930863$	ohm
$l_2/v = 0.2196553$	second	$z_2 = 3.5$	ohms
$l_3/v = 1.047$	seconds	$z_3 = 0.3901989$	ohm
$l_4/v = 0.1899826$	second	$z_4 = 3.5$	ohms
$l_5/v = 0.775$	second	$z_5 = 0.3310432$	ohm
$l_6/v = 0.1429601$	second	$z_6 = 3.5$	ohms
$l_7/v = 0.4135$	second	$z_7 = 0.2752242$	ohm
$l_8/v = 0.06693098$	second	$z_8 = 3.5$	ohms
$l_9/v = 0.106$	second	$z_9 = 0.2968349$	ohm.

To realize the pole locations (27), the values of S_1, S_3, S_5, S_7, S_9 in (9) were shifted from Butterworth positions to desired positions in small steps. Every time a step toward the desired poles is made, element values compatible with the new poles are found by solving (9) anew. While this is accomplished, the lengths of the stubs responsible for the desired transmission zeros are not allowed to change. Finally, the element values that attain the following dominant poles:

$$-0.116232 \pm j\,1.001723$$
$$-0.381702 \pm j\,1.008266$$
$$-0.750730 \pm j\,0.966249$$
$$-1.240295 \pm j\,0.696060$$
$$-1.534504 \pm j\,0.0.$$

and the transmission zeros (28) are

$l_1/v = 1.19$	seconds	$z_1 = 1.119369$	ohms
$l_2/v = 0.250467$	second	$z_2 = 3.931361$	ohms
$l_3/v = 1.047$	seconds	$z_3 = 0.3983251$	ohm
$l_4/v = 0.1863467$	second	$z_4 = 3.643610$	ohms
$l_5/v = 0.775$	second	$z_5 = 0.3439001$	ohm
$l_6/v = 0.1320657$	second	$z_6 = 3.587569$	ohms
$l_7/v = 0.4135$	second	$z_7 = 0.2936443$	ohm
$l_8/v = 0.06099037$	second	$z_8 = 3.564882$	ohms
$l_9/v = 0.106$	second	$z_9 = 0.3144931$	ohm.

The attenuation characteristic of the network characterized by the above element values is shown by the dashed curve in Fig. 8.

The above example illustrates the design of a distributed network from a lumped design that has eight finite transmission zeros and nine transmission poles. The distributed network which was designed to have Butterworth poles as its

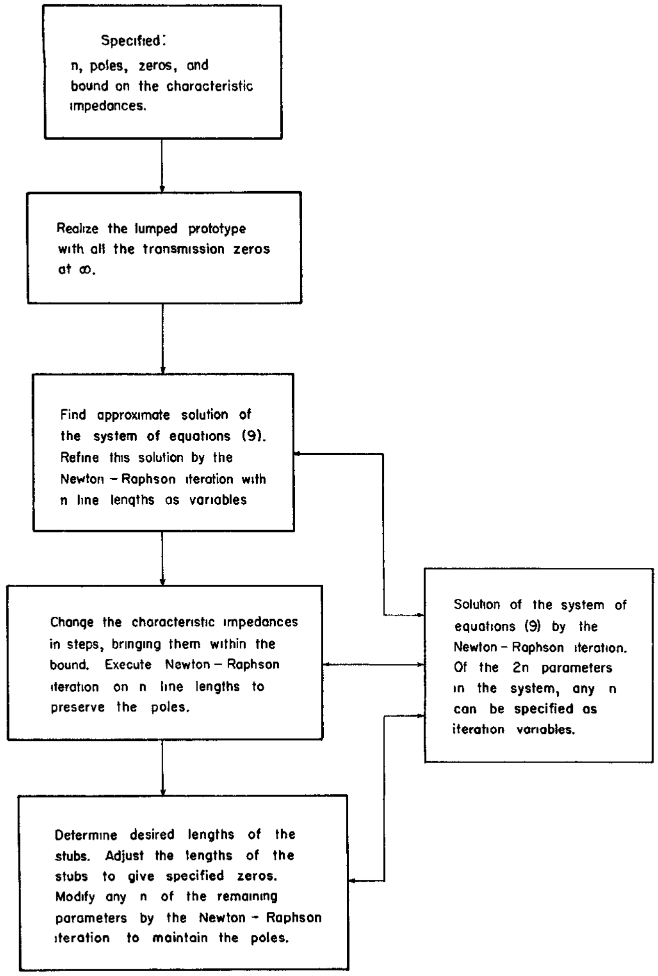


Fig. 9. The synthesis procedure outlined in Sections IV, V, and VI, summarized.

dominant transmission poles was used as the starting point and following the realization of transmission zeros, the transmission poles were shifted from the Butterworth locations to the desired locations. An alternate approach would be to design the lumped network having the desired transmission poles and all the zeros at infinity. Then, following the procedure outlined earlier in this section and in Sections III and IV, a distributed network having the desired transmission poles could be synthesized. The lengths of the stubs would then be adjusted for prescribed transmission zeros. The block-diagram in Fig. 9 summarizes the synthesis procedure outlined in Sections IV, V, and VI.

The transmission delay cuased by the connecting lines combined with the repetitive nature of the transmission zeros produced by the shunt stubs produces transmission poles in addition to the dominant poles which are being controlled. The dip in the attenuation near $\omega = 3.25$ rad/s is caused by such a nondominant pole. It is sometimes possible to approximately nullify the undesirable effect of a nondominant pole by placing opposite it a transmission zero on the real frequency axis. A general strategy for accomplishing this can be formulated as a nonlinear programming problem. This nonlinear programming problem may then be approximately solved by successive linearization, the linear problem thus obtained at each step being solved by simplex algorithm. For this particular example, it is observed that stub (1), in addition to creating a desired transmission zero at $j\,1.32$, places a transmission zero at $j\,3.96$. This latter zero is very close to the desired transmission zero at $j\,3.800$ produced by stub (7). The stub (7) can, therefore, be utilized elsewhere. When the length of the stub (7) was adjusted to place a transmission zero at $j\,3.25$, the changed element values were found to be

$l_1/v = 1.19$	seconds	$z_1 = 0.9373091$	ohm
$l_2/v = 0.250467$	second	$z_2 = 3.609804$	ohms
$l_3/v = 1.047$	seconds	$z_3 = 0.4040285$	ohm
$l_4/v = 0.1863467$	second	$z_4 = 3.544972$	ohms
$l_5/v = 0.775$	second	$z_5 = 0.3478956$	ohm
$l_6/v = 0.1320657$	second	$z_6 = 3.51709$	ohms
$l_7/v = 0.4835$	second	$z_7 = 0.3755778$	ohm
$l_8/v = 0.1$	second	$z_8 = 1.976803$	ohms
$l_9/v = 0.106$	second	$z_9 = 0.3387325$	ohm.

The attenuation of the filter after this adjustment is shown by the solid curve in Fig. 8. This filter was constructed in strip lines and the measured attenuation is indicated by crosses in Fig. 8.

VII. Conclusion

An iterative dominant-pole synthesis technique for transmission line networks has been presented. It has been demonstrated that synthesis can be performed iteratively with each step in the iteration involving the solution of a system of nonlinear equations. The element values are continuously changed to achieve specified poles and zeros which in turn are perturbed until they meet the prescribed network characteristics. This design procedure can take into account transmission zeros near the passband. Extra degrees of freedom in the design equations, and the fact that a specified number of transmission zeros are not uniquely synthesized by an equal number of stubs, result in a nonunique distributed network for a prescribed set of dominant transmission poles and zeros. Thus a variety of optimization possibilities are left open to the circuit designer.

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